



Predicting Relative-Permeability Curves Directly From Rock Images

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Darcy's law extension for 2-phase flow

“...the obvious futility of attempting to solve these [Navier-Stokes] equations for multiply connected passages composed of the pores of a porous medium has necessitated a direct empirical solution of the problem.”

M.Muskat and M.Meres *The flow of heterogeneous fluids through porous media* Physics, 1936

Pore-network model

“The use of high speed computers, such as Illiac, will make possible tests of network model that I could not even consider...”

I.Fatt, *The network model of porous media*
Transactions AIME 1951

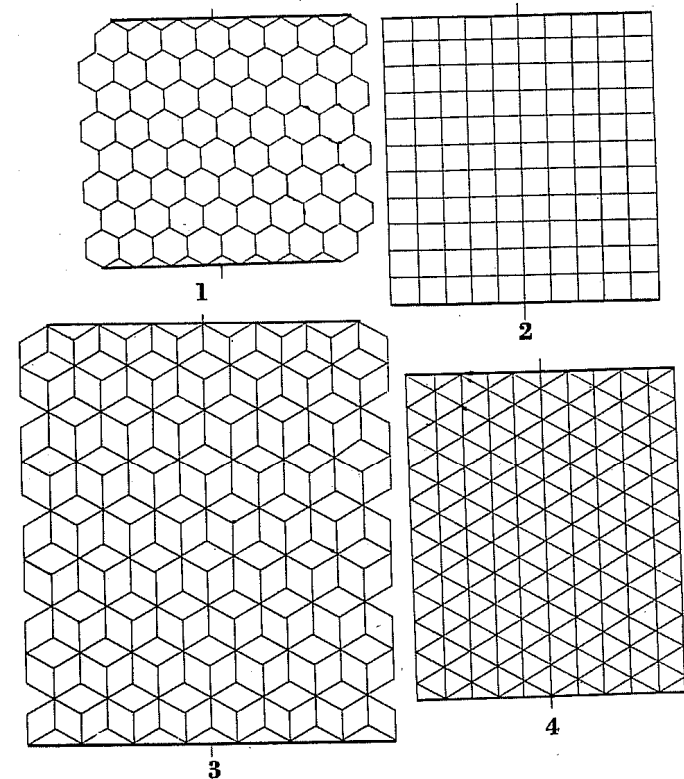
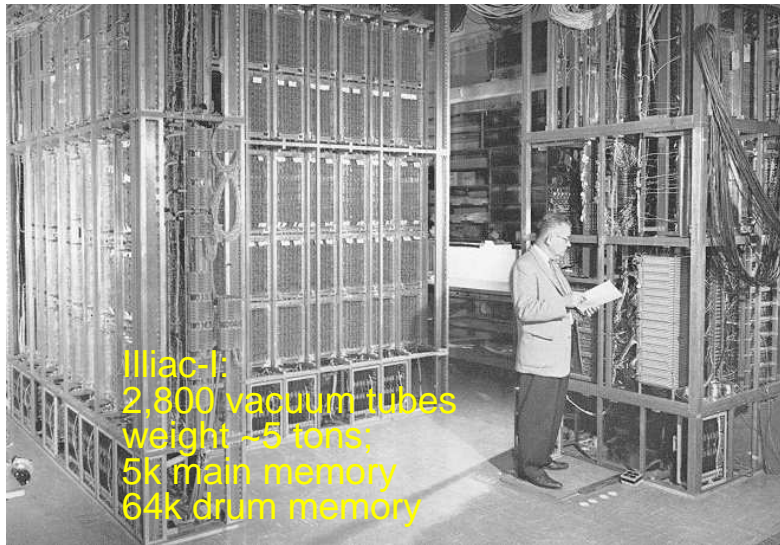
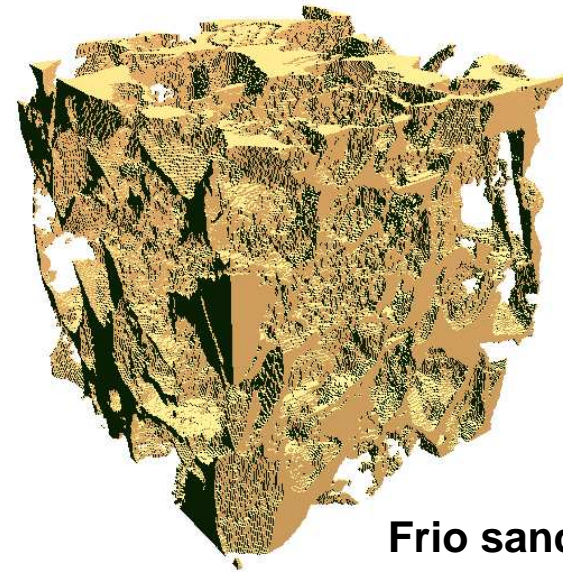


FIG. 1—SINGLE HEXAGONAL NETWORK.
FIG. 2—SQUARE NETWORK.
FIG. 3—DOUBLE HEXAGONAL NETWORK.
FIG. 4—TRIPLE HEXAGONAL NETWORK.

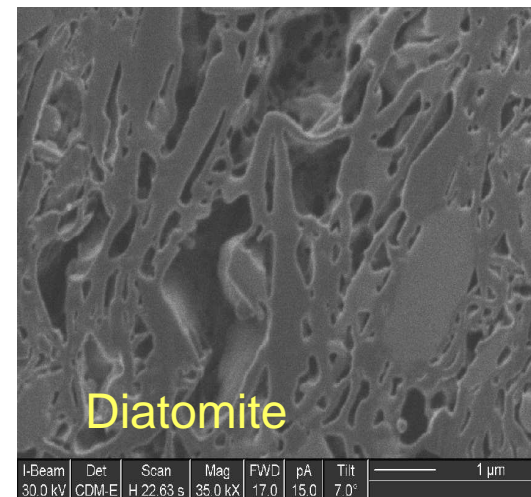
New opportunities

3D reconstruction of the pore space

- X-ray computer tomography (CT): micron-scale resolution
- Focused-ion-beam (FIB): nanometer scale
- Computing power



Frio sandstone

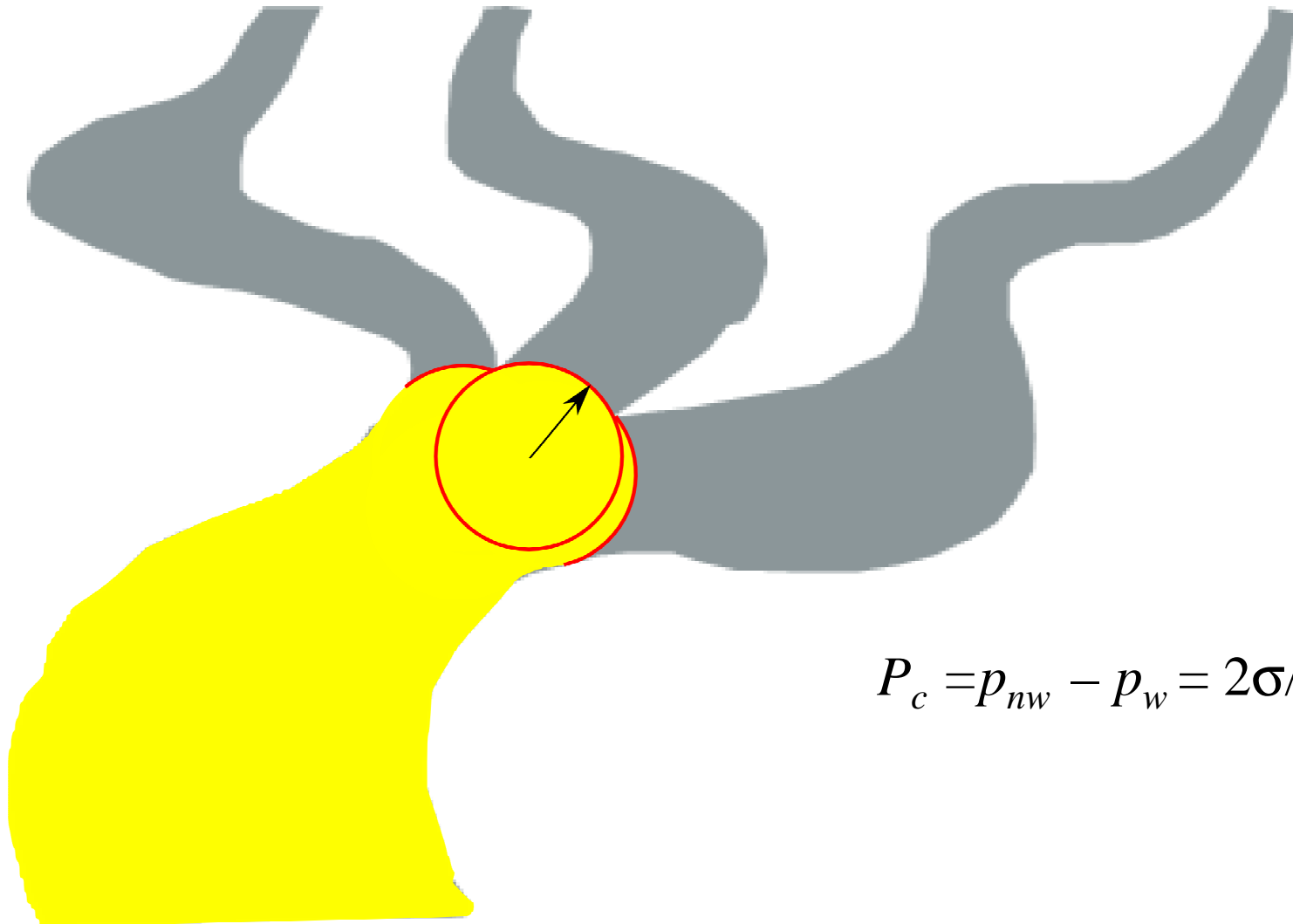


Diatomite

Outline

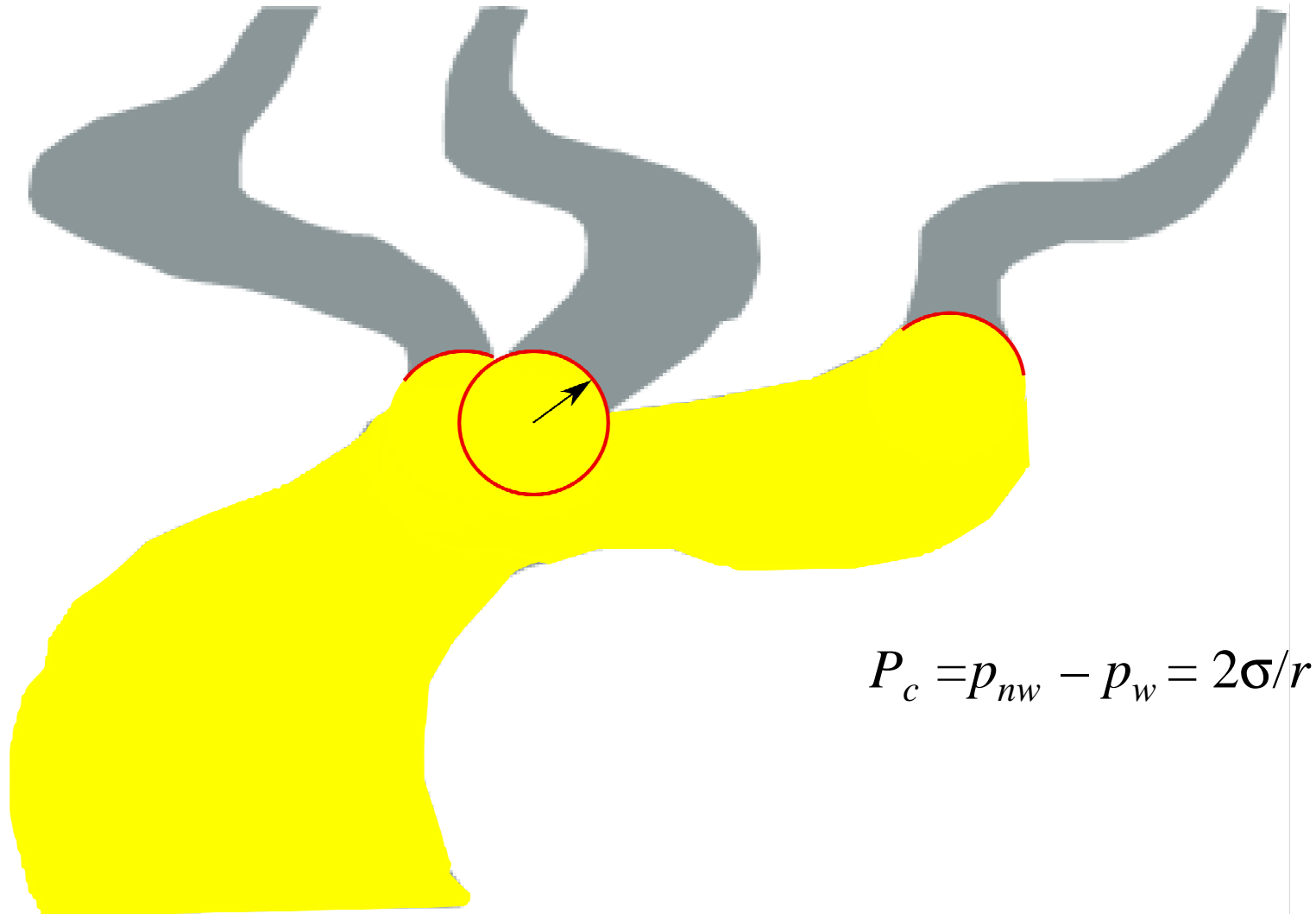
- Maximal Inscribed Spheres method
 - Capillary equilibrium
 - Verification
- Solving flow equations
 - The method of artificial compressibility
 - The curse of dimensionality: domain partitioning
- Relative permeability curves

Maximal inscribed spheres: fluid distribution

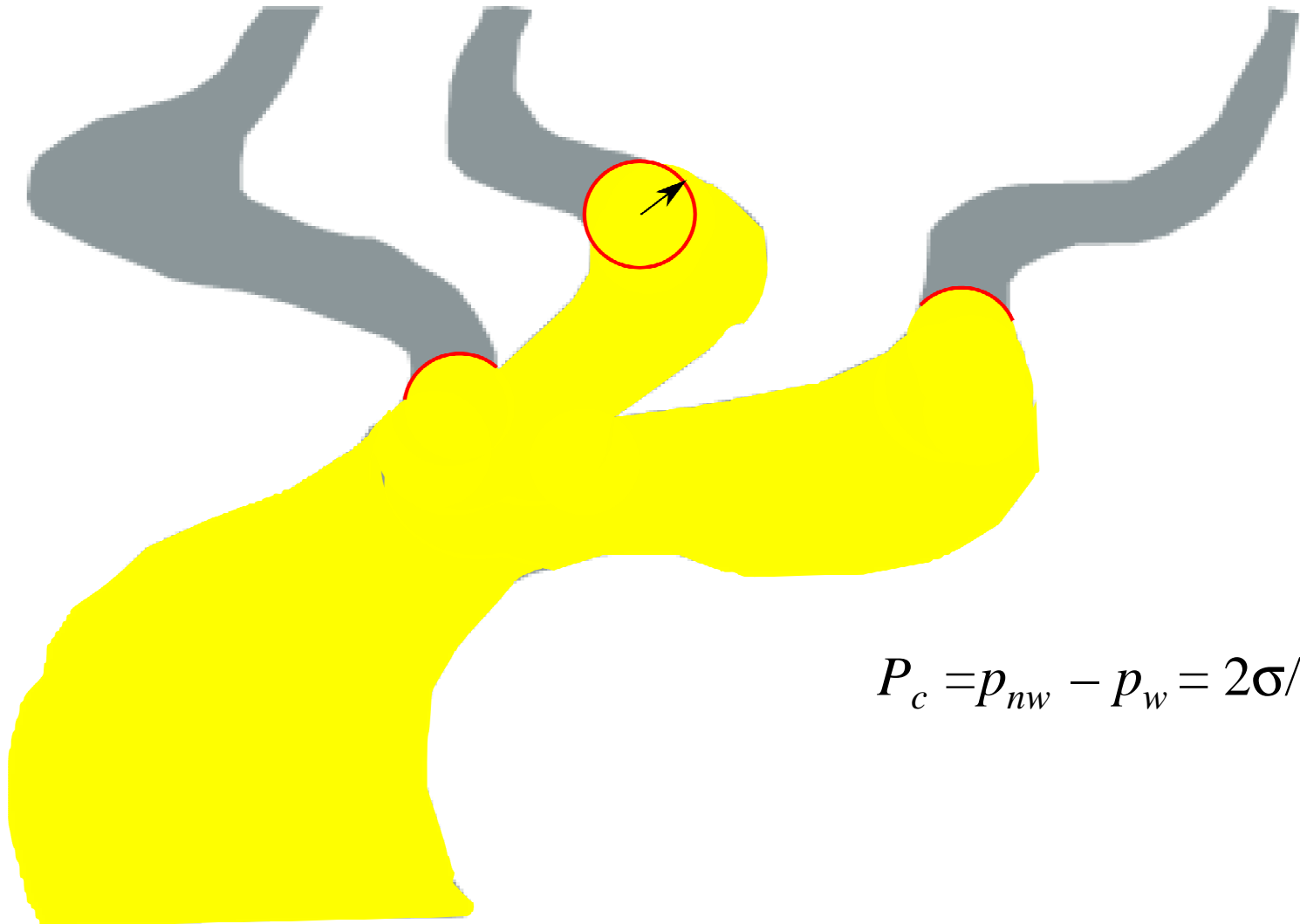


$$P_c = p_{nw} - p_w = 2\sigma/r$$

Maximal inscribed spheres: fluid distribution



Maximal inscribed spheres: fluid distribution

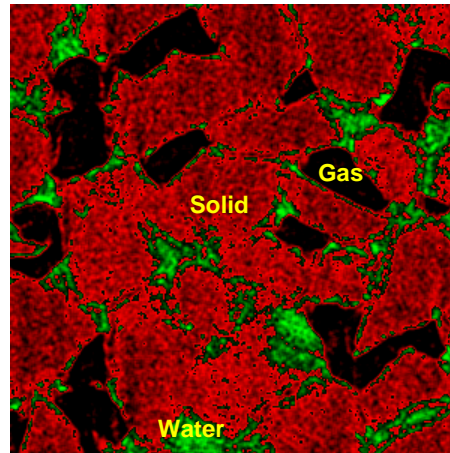


Maximal inscribed spheres

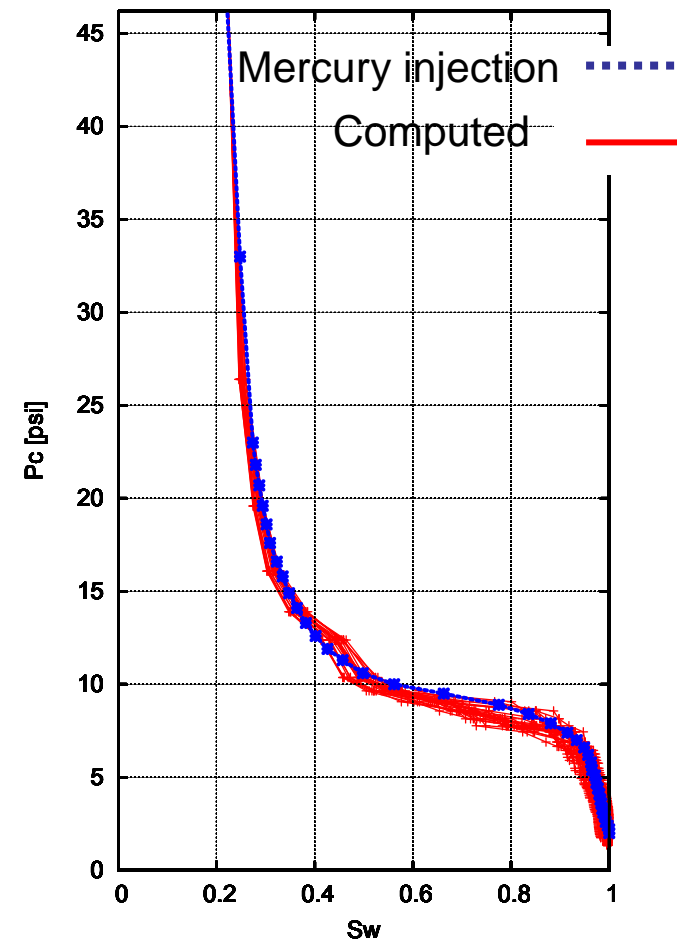
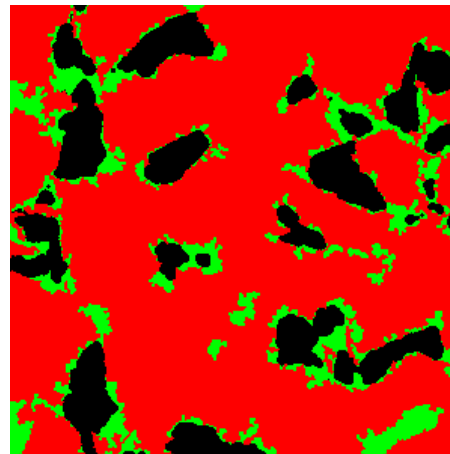
- Assumptions
 - Capillary equilibrium
 - Disperse saturation is negligible
- Implementation
 - Input: 3d binary image of the pore space
 - 3D maximal radii table
 - Connectivity: cluster search
 - Large input data sets

Maximal inscribed spheres: verification

CT image



MIS Calculations



The method of artificial compressibility

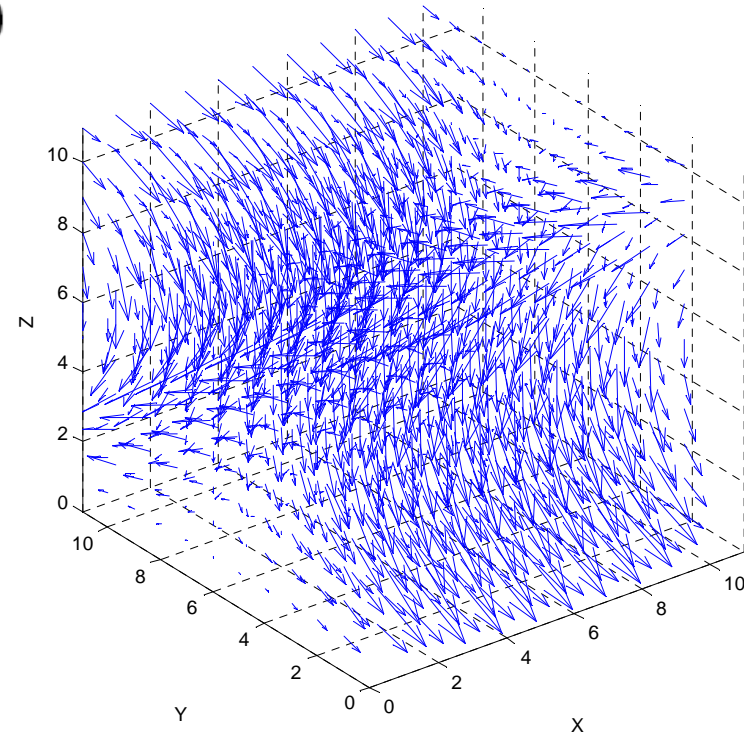
A.Chorin: Journal of Computational Physics 1967

$$\frac{\partial}{\partial t} \mathbf{v} + \text{Re}(\mathbf{v} \cdot \nabla) \mathbf{v} = -\nabla p + \nabla^2 \mathbf{v}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{v} = 0$$

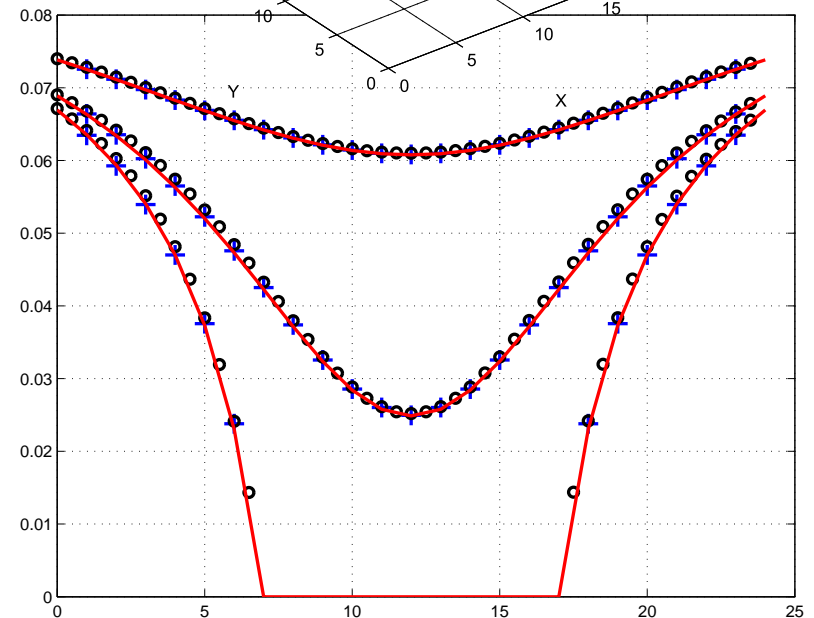
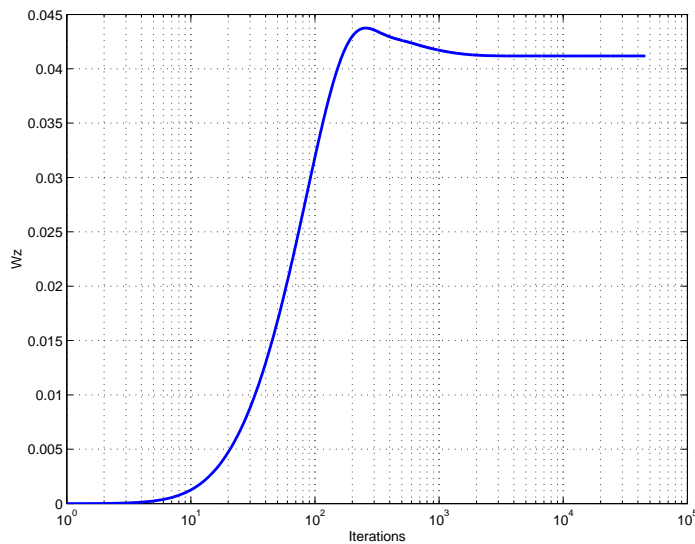
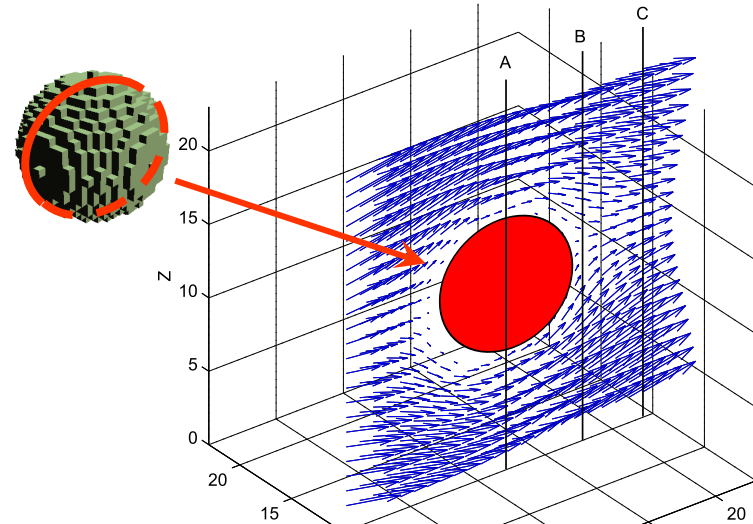
$$\rho = \delta p$$

Computational parameters:
iteration parameter
artificial density
artificial compressibility



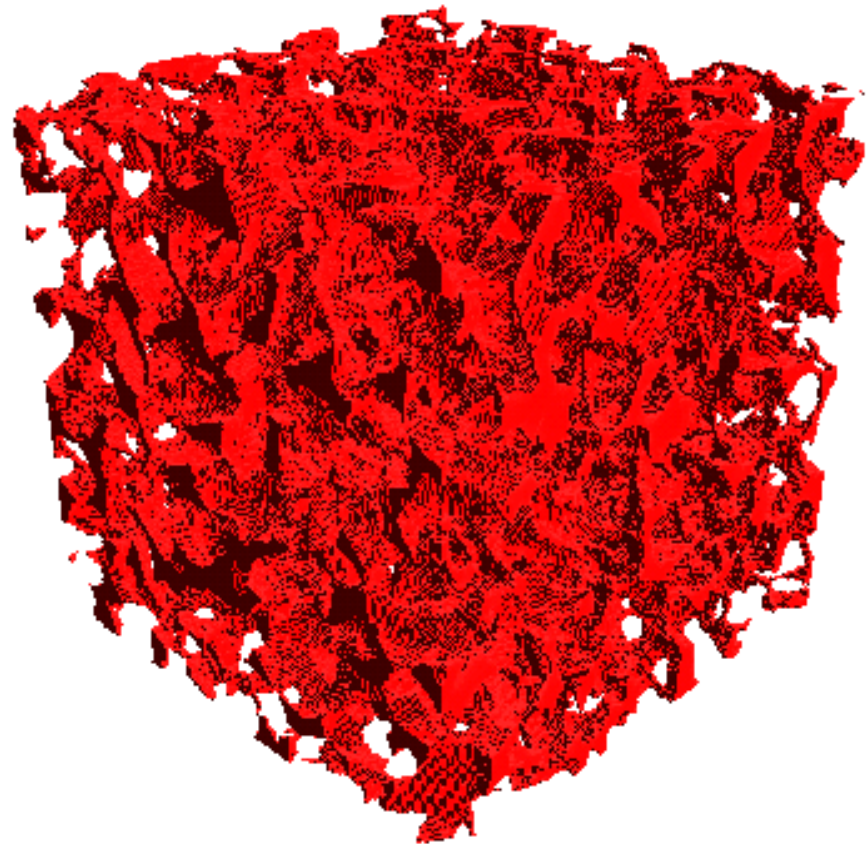
Example: Stokes flow

- Convergence is critically important for estimation of Darcy velocity
- A severe criterion must be imposed

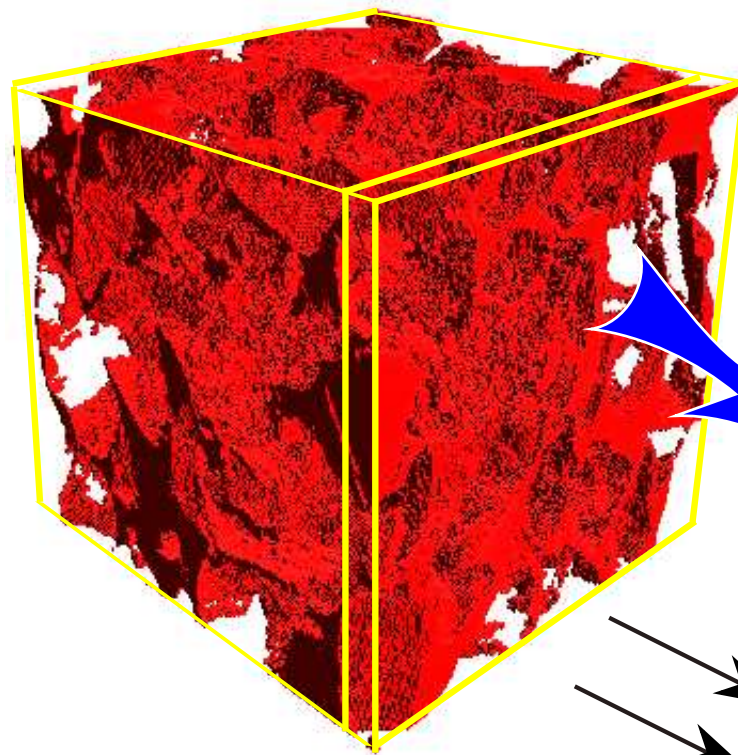


“The Curse of Dimensionality”

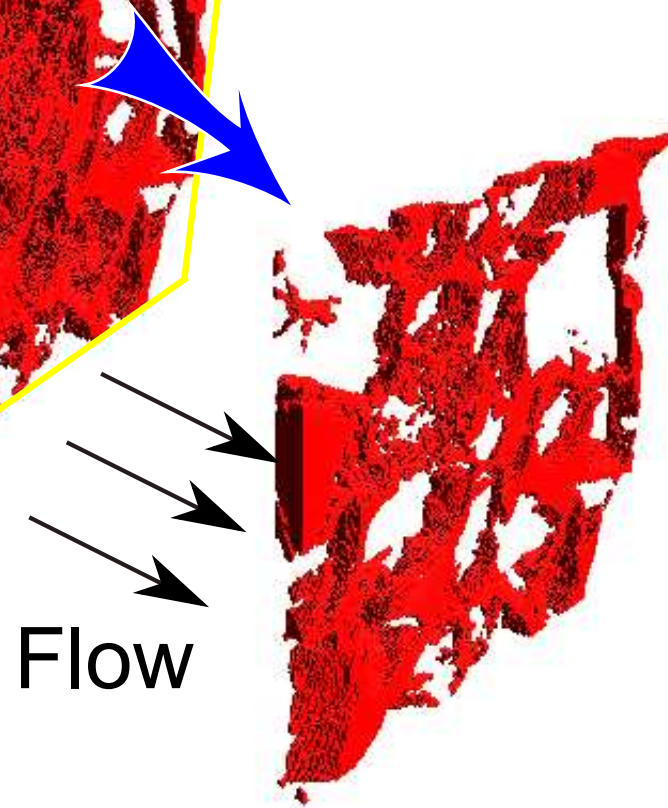
- Good news:
 - High-resolution images → accurate discretization
- Bad news:
 - $5\text{mm} \times 5\text{mm} \times 5\text{mm}$ at 5 micron resolution = 1GB
 - Millions of grid cells → enormous volume of computations



Slicing the whole sample into layers

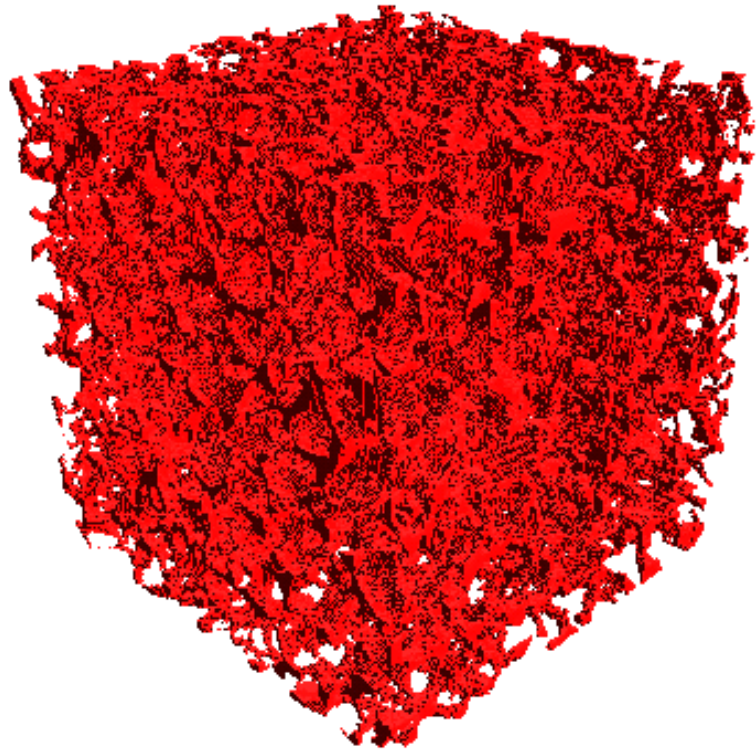


Workaround:
Processing the whole
image by parts



Evaluation of relative permeability curves

Wetting fluid

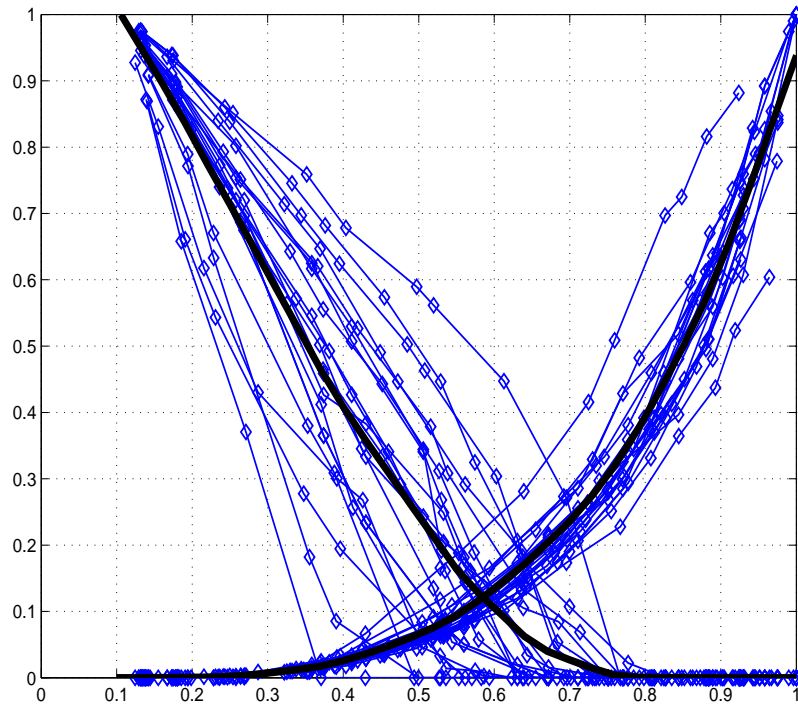


Non-wetting fluid



MIS-calculated fluid distribution

Relative permeability evaluation



MIS-calculations

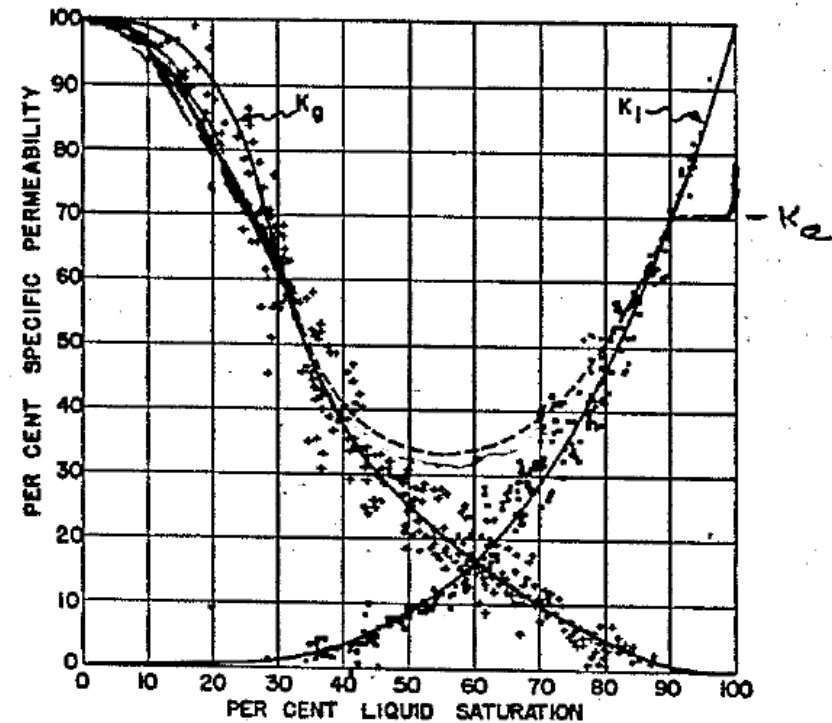


FIG. 9. Composite curves showing permeability-saturation data for all four sands.

Wyckoff & Botset 1936

Summary and conclusions

MIS-calculations + FD simulations:

- Benefits:
 - Partitioning of a large sample → distributed computations
 - Analysis of the digital image bypassing pore network extraction
 - Large high-resolution images can be processed on a desktop computer
- Weaknesses
 - Severe criterion for convergence
 - “The curse of dimensionality”
- A promising approach for routine applications
- Generalizations
 - Modeling of the impact of mechanical deformation, mineralization, etc

Acknowledgments

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